Continous variable flavor number scheme for DIS modelisation

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Abstract. DGLAP equations are modified in order to use all the quark families in the full scale range satisfying kinematical constraints and sumrules thus having complete continuity for the pdfs and observables. Some consequences of this new approach are shown.

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INTRODUCTION

In the present approaches pdf evolution is performed with a fixed number of quark species (ranging from three to six). Usually at some arbitrary fixed values the number of quark flavors is increased by one and the pdfs are rearranged according to theory inspired rules before resuming evolution.

The path of this work is different: it imposes continuity on the observables and pdfs, heavy quark kinematical constraints,momentum sum and quark counting rules and uses always six flavors. For this DGLAP [1] equations are extended to accomodate explicitly heavy quarks.

EXTENDED DGLAP EQUATIONS

They will be written:

$$\frac{\partial g(x,Q^2)}{\partial ln(Q^2)} = \mathcal{P}_{gg} \otimes g(x,Q^2) + \sum_{q=d}^{t} \mathcal{P}_{gq} \otimes q^+(x,Q^2) \quad (1)$$

$$\frac{\partial q^+(x,Q^2)}{\partial ln(Q^2)} = \delta_q \otimes \left[\tilde{\mathcal{P}}_{qg} \otimes g(x,Q^2) + \sum_{r=d}^{t} \tilde{\mathcal{P}}_S^{+} \otimes r^+(x,Q^2) \right] + \mathcal{P}_{NS}^+ \otimes q^+(x,Q^2) \quad (2)$$

$\otimes$ notes the convolution product.
$g(x,Q^2)$ is the gluon pdf at the kinematical variables x,Q^2.
q^+ is the sum of quark q and antiquark $\bar{q}$ pdfs.
$\mathcal{P}$ are the splitting functions which also depend of x,Q^2.[1]

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[1] *Notations are [2] inspired.*

$l_q = (1 + \frac{4m_q^2}{Q^2})^{-1}$ is the kinematic upper limit of x for a quark q of mass m_q.

$\delta_q = \delta(x - l_q)$ effect is identical to the replacement of x by $\xi_q = l_q^{-1}x$ the so-called rescaling variable. It can also be considered as transforming the splitting functions in a way which satisfy the quark kinematical constraint.

- $\delta_q \otimes$ is the only change in DGLAP. For light quarks it is a do nothing operation.
- For $Q^2 \to \infty$ the usual equations are restored.
- For $Q^2 \to 0$ the heavy quark equations decouple due to the kinematical factor as requested by Appelquist-Carazzone theorem.
- $q^- = q - \bar{q}$ pdfs are subject to the same kind of equations (but with no gluon) and have exactly the same extension which will however be in action (and very weakly) starting only at NNLO order in α_s. They will not be considered any further.
- Coefficient functions which are related to the pdfs with the same kind of linear structure will be extended exactly in the same way.
  In fact it is already the case in the latest so-called schemes.

SUMRULE CONSTRAINTS

The momentum sumrule imposes constraints which have to be satisfied (and are for the standard DGLAP). They are obtained by requesting that the first momentum of the sum of all parton distributions is constant and equal to 1 for any Q^2 value. Taking the Q^2 derivative of this momentum, using the DGLAP equations and the property that the n-momentum of a convolution product is equal to the product of the n-momenta of its components, one find easily:

$$\mathcal{N}_f \mathcal{Q}_{qg} + \mathcal{Q}_{gg} = 0 \tag{3}$$
$$\mathcal{N}_f \mathcal{Q}_S^+ + \mathcal{Q}_{gq} + \mathcal{Q}_{NS}^+ = 0 \tag{4}$$

Where $\mathcal{Q} = \int_0^1 \mathcal{P} x dx$ and $\mathcal{N}_f = \sum_{q=d}^t l_q$

$\mathcal{Q}(\mathcal{P})$ being polynomials in flavor number, the only solution is to redefine the latter as being equal to $\mathcal{N}_f$. Quark kinematical limit or longitudinal phase space l_q then may be also viewed as quark activity going smoothly from 0 to 1. To be fully consistent one has to use this new $\mathcal{N}_f$ in the β function driving the α_s renormalisation equation because β_0 appears in both $\mathcal{P}_{gg}$ and β.

THEORETICAL CONSIDERATIONS

Extended DGLAP equations have been set up very close to the ordinary ones.

- They reduce to it when scale Q^2 is far away from any heavy quark mass squared.
- They satisfy the kinematic constraints of the heavy quarks.
- The resulting objects (pdf,derivatives of pdf, charged and neutral currents structure functions ...) are continous in x and Q^2.

- They are supported by works and concepts which are not really new:
    - ξ the scaling variable was used in many papers, see [3] as an example.
    - [3] use also anomalous dimensions variable with Q^2. Anomalous dimensions leading to splitting functions their arguments should hold here. They advocate[2] $l_i \approx (1 + \frac{2m_i^2}{Q^2})^{-1}$.
    - Moreover [3] presented a β_0 variable with Q^2 in the β-function. They advocate here $l_i \approx (1 + \frac{5m_i^2}{Q^2})^{-1}$.
    - [4] have considered $\mathcal{N}_f$ to be Q^2 and even α_s order dependent and there is still developements on the β-function. [5]
    - Small are the differences between massless and continous behavior for α_s.
    - The procedure leading to satisfaction of the kinematical constraints as been used lately for coefficient functions in GM-VFNS schemes [7].
    - Pesented here is the simplest solution to the chosen goal but it is still possible to make modifications. An example is to multiply δ_q by $\phi(Q^2)$ a positive non decreasing continous function equal to 1 for $Q^2 \to \infty$. An other possibility is to modify the $\mathcal{P}$ coherently and differently for different α_s and $\mathcal{N}_f$ orders following [4] example.

Satisfiying all the points it was designed for, it as some serious advantages:

- It does not mix up different α_s orders as do mixed schemes.
- Heavy quarks participate to the evolution when they start to appear, that is at the beginning and at very low x and even at leading order in α_s.
- There is no internal and external partons, only internals.
- It covers the charm-bottom region where they both open up which is not yet the case in VFNS.
- It should give a better treatement of the small x region where heavy quark pdfs are born first.

It seems that ressummation is done, due to evolution, but not the forward divergence pole subtraction (the two dimensions phase space required by a massive quark being already replaced by the one dimension x). In that case this method should be considered as a phenomenological model which may be optimized by adjusting the ϕ functions with the help of the rates given by [8]. The comparaison between the $F_2^{c\bar{c}}$ of the two approaches would however be not that simple, having so different concepts (quark internal versus external).

An important point is that nature has 6 flavors and not 3,4,5 or 6 depending on the context so neglecting that fact is making an approximation. This paper is also making an approximation which is certainly valid when $\mathcal{N}_f(Q^2)$ is close to an integer (at least in a mathematical world where the heavy quark masses are well sparated and for any

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[2] *they did note have the same name and used it differently however.*

number of flavors)
Maybe renormalisation group theory could bring some light to this.

PDF CFNS MASSLESS COMPARAISON

Figure 1 shows the parton species momentum integral fractions in cfns and massless schemes. Starting evolution at $Q_0^2 = .75 Gev^2$, $\quad g, u_v, d_v, \bar{u}, \bar{d}, 2\bar{s} = k(\bar{u} + \bar{d})$ pdfs are fitted at NNLO using preliminary H1 data and the QCDFIT program [6] with extended DGLAP equations. k is determined by the condition [9].
$\int_0^1 x(s + \bar{s})dx = 0.54 \int_0^1 x(\bar{u} + \bar{d})dx$
Behavior differences are impressive. This justify the idea of evolving heavy quarks even when they have to be considered as heavy.
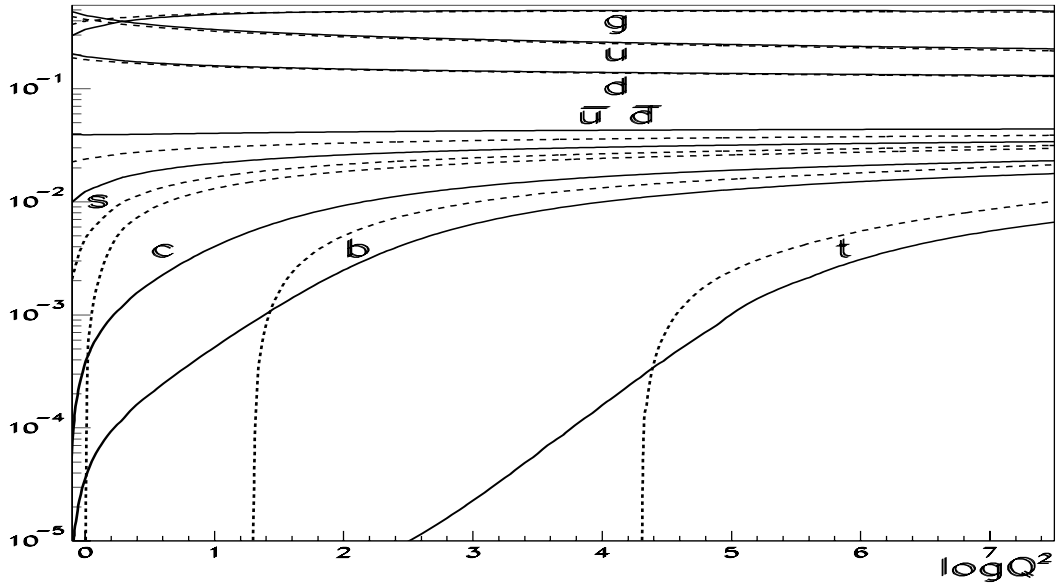

FIGURE 1. Partons momentum fractions cfns:full lines, massless:dotted lines.

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